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SIMPLE HIGH-MOMENTUM NEUTRINO BEAMS

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Summary

For supplying neutrinos to a detector with a large aperture, such as a bubble chamber (e.g., 1 meter radius), intense beams, containing a large fraction of all the pions produced in an energy band of adjustable width, lying in the range from about 60 to 200 GeV/c, can be produced with a single pair of quadrupoles. The beam may be tuned to any central momentum value within this range, and will then pass a wide range of momenta, depending on the system parameters. Above about 200 to 250 GeV/c no focusing system is needed; most of the neutrinos will traverse the detector without further collimation.

Separation of neutrinos from antineutrinos requires additional bending magnets to isolate one sign of charged pions.

Neutrino Beams Without Focusing Elements

The simplest possible high-energy neutrino beam may be obtained by setting a suitable shield some distance downstream (at 0° production angle) from a target, with the neutrino detector behind the shield; the natural relativistic forward collimation of high-momentum pions concentrates a large fraction of the decay neutrinos in the solid angle subtended by the detector.[1] To cite a numerical example: for a detector of 2-meter diameter using a 500-meter decay length plus a 100-meter shield, all pions produced within a cone of half-angle 1/600 = 1.67 mrad will contribute (neglecting a small correction for the pion-neutrino decay angle). Taking the median production angle as 0.4/p for particles of momentum p, we see that pions of 240 BeV/c or above need no additional focusing. At this momentum about half the decay neutrinos are between 40-95 BeV/c, and are emitted at an angle averaging 0.6 mrad to the pion direction. Thus for π-decay neutrinos of 50 BeV/c or above, no special focusing devices are needed. The same argument applies to K-decay neutrinos at a somewhat higher momentum.

Single Quadrupole-Pair Lens

For pions below 240 BeV/c, the median production angles are larger than 1.67 mrad. A quadrupole-lens pair can be used to focus any particular momentum into a parallel beam; then for some momentum band in the vicinity of the focused value, the divergence of the pions will still allow a sufficiently small neutrino image at the detector. The lens pair will not discriminate in sign; auxiliary bending magnets are needed for that. We now investigate the range of conveniently available values.

[1] This was suggested by A. L. Read.

The available parameters of the lens are the focal length and the f-stop (more conveniently defined here by the maximum production angle accepted). We use a single mean value for the vertical and horizontal acceptance angles, and assume thin-lens optics. By varying the focal length of the lens, f, either a real or virtual image is formed from which the diverging beam just reaches the prescribed image diameter (see Fig. 1). Outside these two settings, the image exceeds the prescribed size. The momenta corresponding to the focal lengths so calculated then define the useful transmission region of the lens. From elementary geometrical optics, the two image distances v_+ and v_- for which this occurs, are given, if the lens radius is a and the image radius R, by

$$v_\pm = \frac{(D - u)a}{R \pm a} \qquad (1)$$

The corresponding focal length $f_\pm$ of the lens, (which is u for the central momentum) is then given by $f_\pm = 1/(1/u \pm 1/v_\pm)$.

In the thin-lens approximation, the focal length of a pair of quadrupoles of equal length is related to the momentum by

$$f = \frac{a^2 p^2}{.09 B_o^2 \ell^2 (2\ell/3 + d)} \qquad (2)$$

where

f = focal length,

a = equivalent lens aperture radius,

B_0 = maximum magnetic field at pole, in Teslas,

ℓ = length of each quadrupole,

d = space between quadrupoles,

p = particle momentum, BeV/c.

All lengths are in meters. To simplify, we will take $d = \ell/3$, so that $\ell^2(2\ell/3 + d)$ becomes ℓ^3.

As a reasonable initial value, we take D, the source-detector distance, to be 600 meters, reserving about 100 meters for the shielding in front of the detector; and we assume that the acceptable image size at the detector is a 2-meter diameter circle. The problem then becomes the classical optical depth-of-field calculation, and the result is that the acceptable momentum band depends to a very good approximation only on the lens aperture-stop and not on its focal length. If we specify the aperture stop in terms of the half-angle of the cone of acceptance, we can then draw up the following table.

TABLE I. Momentum limits within which particles will be focused within a 1-meter radius circle of confusion, assuming the lens focal length is set to give point-to-parallel focussing at 100 BeV/c.

Acceptance cone half-angle, mrad	Lower momentum limit (BeV/c)	Upper momentum limit
2.0	73.9	272.0
3.3	80.5	140.0
5.0	86.9	123.5
6.6	88.7	114.0
10.0	91.8	108.6

As one might expect, the smaller apertures show the largest depth of field.

Gain

Let us take the angular radius of the Cocconi disk to be 0.4/p mrad, and drastically simplify the problem by assuming the primary distribution to be uniform inside the Cocconi disk, zero outside. The angle subtended by the detector at the source is 1.66 mrad for the values we are considering. The possible gain is limited either by the Cocconi disk or the lens aperture, whichever is smaller. Table II indicates the gain to be expected in charged particle illumination; small aperture lenses are clearly of not much use.

TABLE II. Lens gain as a function of aperture and momentum, for a detector subtending a half-angle of 1.66 mrad.

Particle momentum, BeV/c	Cocconi-disk cone angle, mrad	Illumination gain for lens aperture (half-angle) in mrad 2	3.33	5	6.66	10
240	1.66	1	1	1	1	1
200	2.0	1.44	1.44	1.44	1.44	1.44
120	3.33	1.44	4.0	4.0	4.0	4.0
100	4.0	1.44	4.0	5.8	5.8	5.8
80	5.0	1.44	4.0	9.0	9.0	9.0
65	6.66	1.44	4.0	9.0	16.0	16.0
40	10.0	1.44	4.0	9.0	16.0	36.0

We must also superpose an additional diffusion effect, due to the fact that the neutrino direction is not exactly that of the charged particle. For pions the additional angle has a median value about 3/8 of the Cocconi disk angle so the effect is small; for K-mesons the two angles are about equal. The lens pair affords high gain over a narrow momentum band, when a large aperture is used, and provides also the alternative of moderate gain over a wider momentum band by increasing the distance from source to lens.

Variation of Detector Distance

As might be expected, increasing the source-detector distance or decreasing the required image size decreases the effectiveness of a lens of a given size unless the angular diameter of the allowable circle of confusion--i.e., the detector aperture--is kept constant. For a given detector area, a short distance improves the intensity, the increased illumination more than compensating for the decrease in decay path.

The applicability of the lens pair is thus greatest in the region between about 50 and 150 BeV/c. Above this, the potential gain is small; below 50 one can make significantly better transport systems. The neutrino spectrum of the pions in this range will extend usefully from about 10 to 70 BeV/c.

Lens Strength

The greatest lens strength is needed for the longest source-lens distance to be used. If p, for example, is doubled, then to keep the same acceptance of the Cocconi disk we must double f; then from (2) the length must increase as $p^{1/3}$. The length of the lens is determined by the highest momentum it is desired to focus, which depends on the solid angle subtended at the source by the detector aperture.

The true solid-angle acceptance of a quadrupole pair is about half the geometrical one, and the acceptances are very different in the FD and DF planes. One can either use somewhat larger lens pairs to achieve a given angular acceptance, or adopt the more efficient elliptical-aperture quadrupole. A lens triplet has more nearly equal FD and DF acceptances than a doublet, but no better geometrical efficiency.

Separation of Neutrinos from Antineutrinos

An achromatic bending system between the source and the lens pair can be used to select either positive or negative charged particles. This adds to the complexity of the system, and involves bending magnets in a high particle-flux region.

Unsuitability for Muons

This kind of beam is unsuited for muon collection and use, since the beam divergence is too large. It is assumed that the muons as well as all hadrons in the beam are absorbed or deflected by the shield in front of the detector.

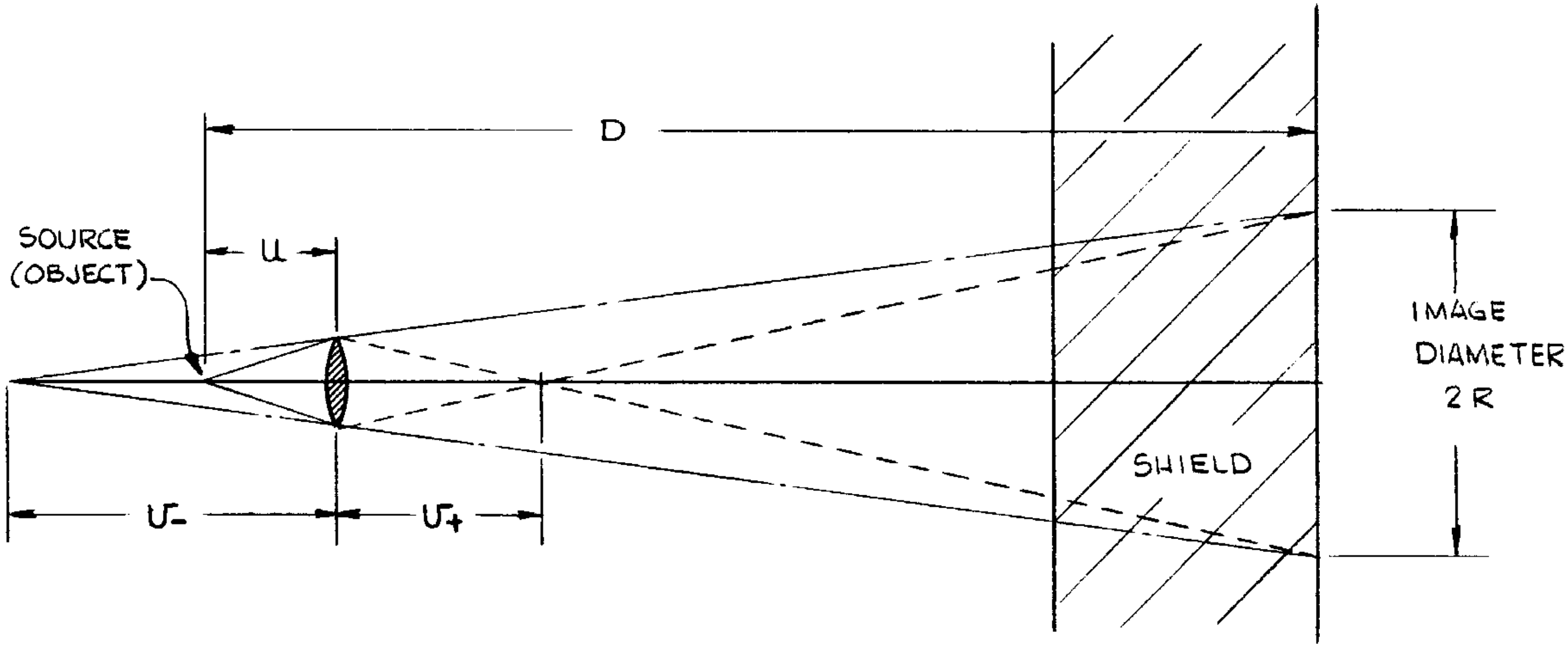

Fig. 1. Single-lens neutrino beam. The object distance is u, and the lens produces a parallel beam at the central momentum. The real and virtual images v_+ and v_- define the limiting acceptances, at which the image just fills the allowed diameter at the detector.